COUPLING BETWEEN HYBRID MODE DIELECTRIC RESONATORS*

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ABSTRACT

Methods for the rigorous numerical calculation of the coupling coefficient between dual hybrid mode dielectric resonators in a circular waveguide are developed. A simplified model for the approximate calculation of the coupling is also presented. Experimental measurements performed to verify the models are found to be in excellent agreements with the calculations.

1. INTRODUCTION

Coupling between dielectric resonators excited in axially symmetric modes ($TE_{01\delta}$, TM_{010}, ..., etc.) has been treated extensively in the literature [1]-[4]. Although some generalized approximate methodology that could be used for coupling computation between non-axially symmetric modes in dielectric resonators has been presented [5], there has been no rigorous treatment of the subject, with results that can directly be used for practical applications. The exception to that is the treatment for coupling of hybrid HE_{11} modes by iris presented in [6].

This paper presents a rigorous technique for coupling calculation between non-axially symmetric modes in cylindrical dielectric resonators enclosed in circular waveguides. A simplified approximate model is derived from the results of the rigorous analysis, which can be used for the design of the resonators without recourse to the detailed numerical solution of the boundary value problems. Experimental measurements performed to verify the models are presented and show excellent agreement with the theoretical calculations.

2. METHOD OF COUPLING CALCULATION

Consider two identical circular cylindrical dielectric resonators of radius a, and length t, placed coaxially inside a perfectly conducting cylinder of radius b as shown in Fig. 1. The resonators have relative dielectric constant ϵ_r and spaced a distance 2ℓ apart. The planar end walls of the enclosure are perfectly conducting and are at distance L to each from the resonators ends.

The model used in [1] to calculate the coupling between the resonators excited in $TE_{01\delta}$ mode approximates each of the resonators by an axial magnetic dipole. One of the dipoles radiates in the waveguide beyond cut off. The fraction of the energy received by the other dipole is used as a measure of the coupling coefficient.

For hybrid mode excitations, it is possible to use a similar approximation. However, in this paper the coupling is computed based on the solution of the boundary value problem for the fields and resonant frequencies in the combined two resonator structure. Coupling calculations for a given mode can be reduced to the calculation of two resonant frequencies: f_e and f_m, which are the resonant frequencies of one resonator with the symmetry plane (shown in Fig. 1) replaced by an electric (perfect conductor) and magnetic (perfect magnetic conductor) walls respectively. Once f_e and f_m are known, the coupling k between the two resonators is given by:

$$k = \frac{f_e^2 - f_m^2}{f_e^2 + f_m^2} \qquad (1)$$

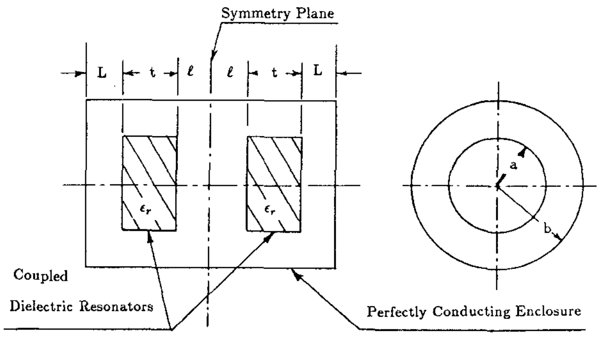

Fig. 1. Two coupled dielectric resonators in a perfectly conducting cylindrical enclosure.

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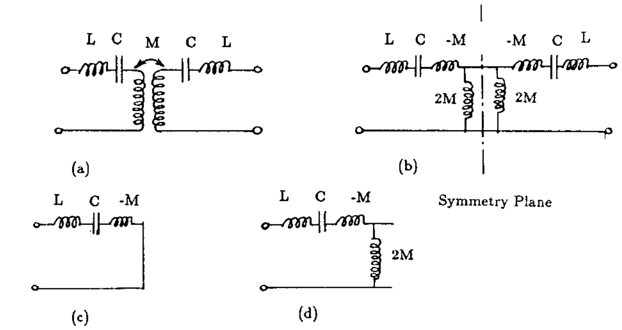
Equation (1) is easily derived from the equivalent circuits shown in Fig. 2. To calculate f_e and f_m, the mode matching method [7] is used. In this method, the fields within each of the end regions of the single resonator are expressed as a linear combination of normal waveguide modes (TE_{mn} and TM_{mn} modes) which satisfy the boundary conditions at the end planes of vanishing tangential electric or magnetic fields. The fields in the dielectric loaded region are represented as a linear combination of hybrid modes of a dielectric loaded waveguide. The azimuthal field variations in both regions must be the same ($\sin m\phi$ and $\cos m\phi$). Application of the appropriate boundary conditions of the continuity of the tangential electric and magnetic fields at the dielectric resonator ends results in a system of linear homogeneous equations in the mode expansion coefficients. The frequencies f_e and f_m are the roots of the equation resulting from equating the systems determinant to zero.



$$f_e = \frac{1}{2\pi\sqrt{(L-M)C}} \qquad f_m = \frac{1}{2\pi\sqrt{(L+M)C}}$$

$$k = \frac{M}{L} = \frac{f_e^2 - f_m^2}{f_e^2 + f_m^2}$$

Fig. 2. Equivalent circuit of two computed resonators (a) & (b)
Symmetry plane replaced by electric short circuit (perfect conductor) (c)
Symmetry plane replaced by open circuit (magnetic wall) (d).

3. NUMERICAL AND EXPERIMENTAL RESULTS

The method outlined above for calculating f_e and f_m was implemented numerically by adopting computer programs developed in Reference [7]. Fig. 3 shows the results of the resonant frequencies for the two lowest hybrid modes. Parameters used in the generation of this figure are: ϵ_r =35.74, a=0.34", b=0.50", t=0.30" and L=0.2". Also shown in Fig. 3 are measured resonances of the structure composed of two such resonators. The agreement between both theory and measurements are quite good. Measured and computed coupling coefficient k as a function of half the separation between the resonators ℓ, as calculated from equation (1) are shown in Fig. 4.

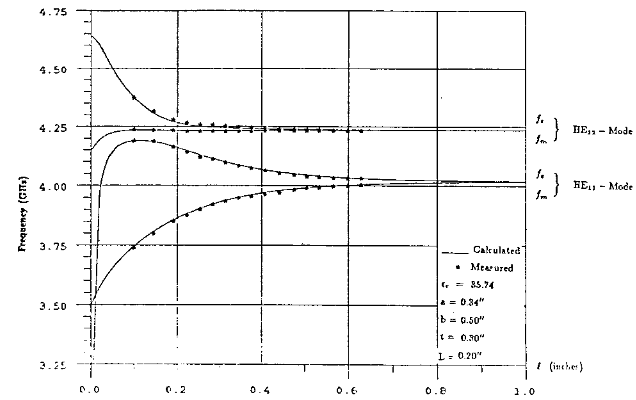


Fig. 3. Calculated and measured results for f_e and f_m of the two lowest hybrid modes.

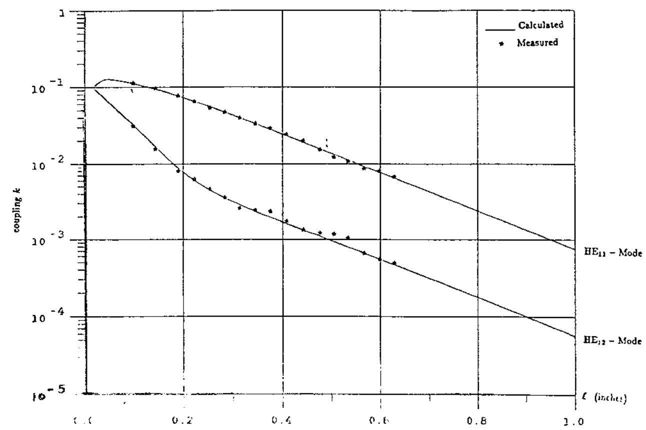


Fig. 4. Calculated and measured coupling k between two resonators for the two lowest hybrid modes.

Coupling was computed for a wide range of parameters of the resonators, the enclosure and the spacing 2ℓ between the resonators, for the lowest frequency hybrid mode (HE_{11}). Few results of these calculations are summarized in Fig. 5. From Fig. 5 it is seen that for values of the coupling k less than .075, the coupling can be accurately described by the expression:

$$k = k_0 \, e^{-2\alpha\ell} \tag{2}$$

where α is the attenuation constant of the TE_{11} mode in a circular waveguide of radius b at the resonant frequency of the resonator, and k_0 is a constant that depends on the resonator parameters. Table 1 gives a typical range of parameters for two coupled values of α and k_0 determined from numerical computations are shown. The table also compares the attenuation factors α as determined from the least squares fit of the computed points, and from the waveguide attenuation constant $\alpha_{W.G.}$ given by:

Table 1. Typical Parameters for Coupled Resonators

a''	b''	t''	ϵ_r	t''	f_0	α	$\alpha_{W.G.}$	k_0
.30	.51	.3	35.74	.2	4.2807	2.8013	2.7995	0.17493
.34	.4	.3	35.74	.2	3.8133	4.1321	4.1313	0.32940
.34	.6	.3	35.74	.2	4.0132	2.2043	2.2042	0.20129
.34	.51	.3	35.74	.2	4.0180	2.9011	2.9011	0.24937
.34	.51	.2	35.74	.2	4.4755	2.7138	2.7125	0.18709
.34	.51	.3	30.0	.2	4.2932	2.7959	2.7951	0.27852
.34	.51	.3	40.0	.2	3.8373	2.9776	2.9771	0.23798
.34	.51	.3	90.0	.2	2.6437	3.3252	3.3249	0.18575
.34	.51	.2	35.74	.1	4.0174	2.9096	2.9096	0.25895
.34	.51	.3	35.74	.4	3.9754	2.9259	2.9254	0.22472
.4	.51	.3	35.74	.2	3.5166	4.1318	4.1321	0.28839

$$\alpha_{W.G.} = \left(\frac{1.841}{b}\right)\sqrt{1-\left(\frac{2\pi b f_0}{1.841c}\right)^2} \qquad (3)$$

where c is the speed of light and f_0 is the resonant frequency obtained as the average of f_e and f_m, with large separation (2ℓ) between the resonators. Agreement between the values of $\alpha_{W.G.}$ and α are excellent (better than 0.06%).

4. CONCLUSIONS

The coupling between hybrid modes in dielectric resonators can be accurately calculated by solving for the resonant frequencies of single resonators with electric and magnetic walls. Experimental results verified the accuracy of the calculations. A simplified exponential model that requires only two parameters k_0 and α is postulated which accurately predicts the results for a limited, but wide range of parameters. The attenuation α is simply the attenuation constant of the TE_{11} mode in a waveguide beyond cut off of the same radius as the enclosure, at the resonant frequency of the resonator.

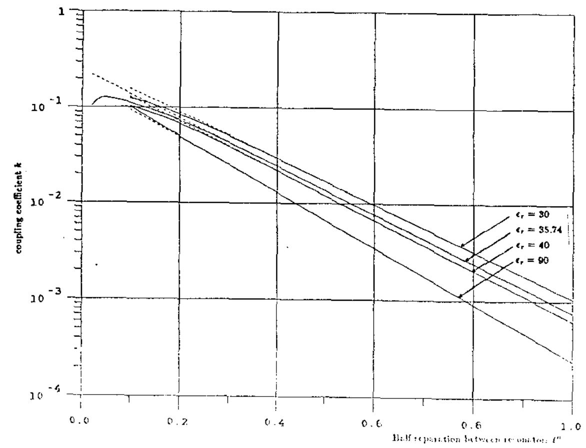


Fig. 5a. Variation of coupling coefficient with separation between resonators for various parameter.

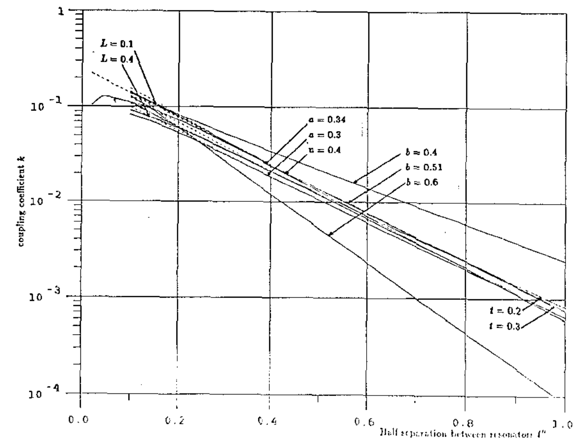


Fig. 5b. Variation of coupling coefficient with separation between resonators for various parameter.

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